Aggregate and Workforce Planning (Huvudplanering)

Production and Inventory Control (MPS)

MIO030

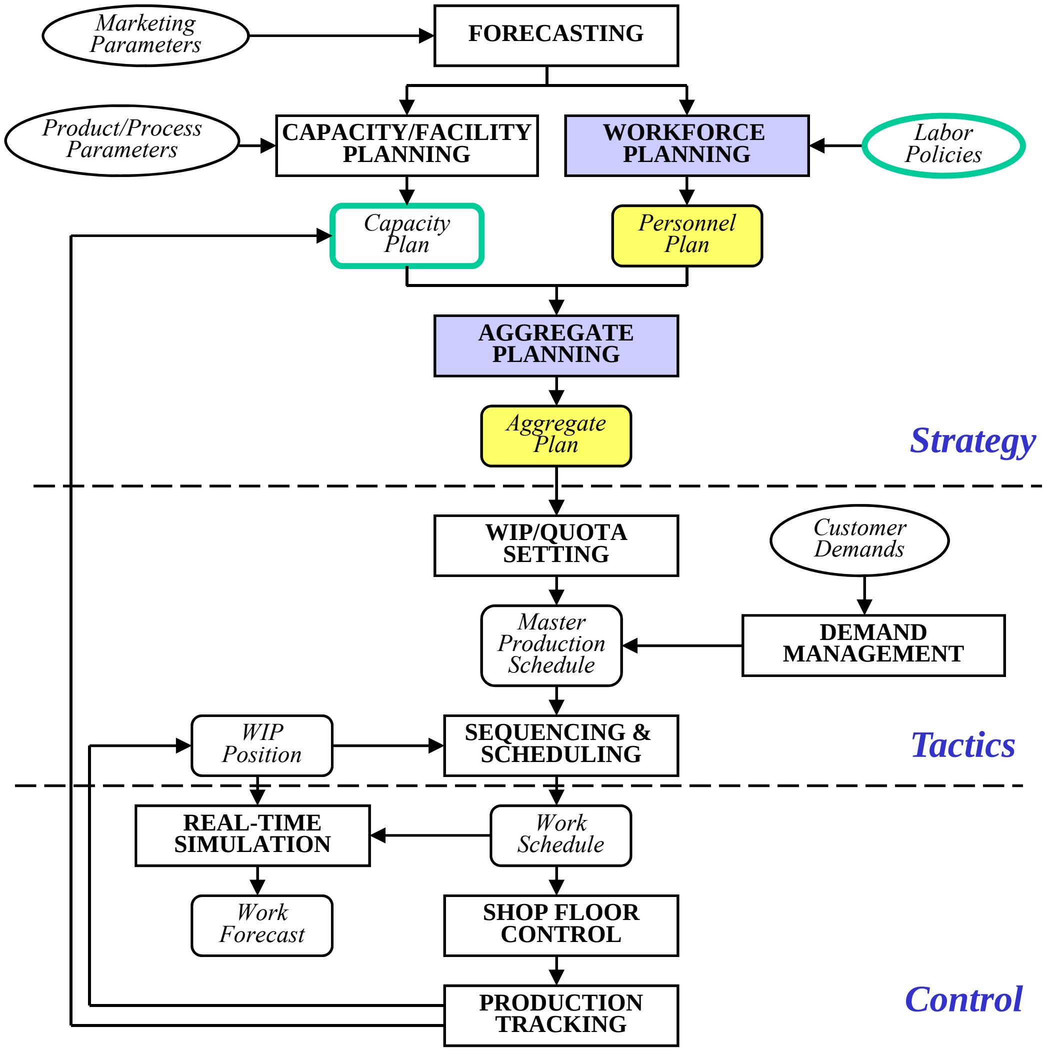
The main reference for this material is the book Factory Physics by W. Hopp and M.L Spearman, McGraw-Hill, 2001.

What is the Role of Aggregate Planning?

- **Role of Aggregate Planning**
  - Long-term planning function
  - Strategic preparation for tactical actions

- **Aggregate Planning Issues**
  - ***Production Smoothing:*** inventory build-ahead
  - ***Product Mix Planning:*** best use of resources
  - ***Staffing***: hiring, firing, training
  - ***Procurement***: supplier contracts for materials, components
  - ***Sub-Contracting***: capacity vendoring
  - ***Marketing***: promotional activities

Aggregate Planning is Long Term



Basic Aggregate Planning Situation

- **Problem:** plan production of single product over planning horizon.
- **Motivation for Study:**
  - mechanics and value of Linear Programming (**LP)** as a tool
  - intuition of production smoothing
- **Inputs:**
  - demand forecast (over planning horizon)
  - capacity constraints
  - unit profit
  - inventory carrying cost rate

A Simple Aggregate Planning Model (I)

Notation:

t = an index of the time periods, $t = 1, \ldots\ldots, \bar{t}$.

d_t = demand in period t.

c_t = capacity in period t.

r = unit profit (not including holding cost)

h = cost to hold one unit of inventory for one period.

X_t = quantity produced during period t.

S_t = quantity sold during period t.

I_t = inventory at the end of period t.

A Simple Aggregate Planning Model (II)

Formulation

summed over planning horizon

$$\max \ \sum_{t=1}^{\bar{t}} rS_t - hI_t$$ *sales revenue - holding cost*

subject to

$S_t \le d_t$	$t = 1,.....\bar{t}$	*demand*
$X_t \le c_t$	$t = 1,.....\bar{t}$	*capacity*
$I_t = I_{t-1} + X_t - S_t,$	$t = 1,.....\bar{t}$	*inventory balance*
$X_t, S_t, I_t \ge 0$	$t = 1,.....\bar{t}$	*non-negativity*

Product Mix Planning (I)

- **Problem:** determine most profitable mix over planning horizon
- **Motivation for Study:**
  - linking marketing/promotion to logistics.
  - Bottleneck identification.
- **Inputs:**
  - demand forecast by product (family?); may be ranges
  - unit hour data
  - capacity constraints
  - unit profit by product
  - holding cost

Product Mix Planning (II)

Verbal Formulation

maximize ***profit***

subject to:

production	$\leq$	***capacity***,	*at all workstations in all periods*
sales	$\leq$	***demand***,	*for all products in all periods*

Note: *we will need some technical constraints to ensure that variables represent reality.*

Product Mix Planning (III)

Notation

i = an index of product, $i = 1,\ldots\ldots, m$

j = an index of workstation, $j = 1,\ldots\ldots, n$

t = an index of period, $t = 1,\ldots\ldots,\bar{t}$

$\bar{d}_{it}$ = maximum demand for product i in period t.

$\underline{d}_{it}$ = minimum sales allowed of product i in period t

a_{ij} = time required on workstation j to produce one unit of product i.

c_{jt} = capacity of workstation j in period t.

r_i = net profit from one unit of product i

h_i = cost to hold one unit of i for one period t.

X_{it} = amount of product i produced in period t

S_{it} = amount of product i sold in period t.

I_{it} = inventory of product i at end of t.

Product Mix Planning (IV)

Mathematical Formulation

$$\max \sum_{t=1}^{\bar{t}} \sum_{i=1}^{m} r_i S_{it} - h_i I_{it}$$ ***(sales revenue - holding cost)***

subject to

$$\underline{d}_{it} \le S_{it} \le \bar{d}_{it} \quad \text{for all } i,t$$ ***(demand)***

$$\sum_{i=1}^{m} a_{ij} X_{it} \le c_{jt} \quad \text{for all } j,t$$ ***(capacity)***

$$I_{it} = I_{i\,t-1} + X_{it} - S_{it}, \quad \text{for all } i,t$$ ***(inventory balance)***

$$X_{it}, S_{it}, I_{it} \ge 0 \quad \text{for all } i,t$$ ***(non-negativity)***

A Product Mix Example (I)

Assumptions:

- two products, *P* and *Q*
- constant weekly demand, cost, capacity, etc.
- ***Objective:*** maximize weekly profit

Data:

Product	**P**	**Q**
Selling price	$90	$100
Raw Material Cost	$45	$40
Max Weekly Sales	100	50
Minutes per unit on Workcenter A	15	10
Minutes per unit on Workcenter B	15	**35**
Minutes per unit on Workcenter C	15	5
Minutes per unit on Workcenter D	**25**	**14**

A Product Mix Example (II)

A Linear Programming (LP) Approach:

Formulation:

$$\max \ 45X_P + 60X_Q - 5000$$

subject to :

$$15X_P + 10X_Q \le 2400$$
$$15X_P + 35X_Q \le 2400$$
$$15X_P + 5X_Q \le 2400$$
$$25X_P + 14X_Q \le 2400$$

Solution: Optimal Objective $= \$557.94$

$$X_P^* = 75.79$$
$$X_Q^* = 36.09$$

Net Weekly Profit : Round solution down (still feasible) to: $X_P^* = 75$

$$X_Q^* = 36$$

To get $\$45 \times 75 + \$60 \times 36 - \$5{,}000 = \$535.$

Extensions to the Basic Product Mix Model (I)

Other Resource Constraints:

Notation:

b_{ij} = units of resource j required per unit of product i

k_{jt} = number of units of resource j available in period t

X_{it} = amount of product i produced in period t

Constraint for Shared Resource *j*: $\sum_{i=1}^{m} b_{ij} X_{it} \leq k_{jt}$

Utilization Matching: Let q represent fraction of rated capacity we are willing to run on resource j.

$$\sum_{i=1}^{m} a_{ij} X_{it} \leq qc_{jt} \text{ for all } j, t$$

Extensions to the Basic Product Mix Model (II)

Backorders:

- Substitute $I_{it} = I_{it}^{+} - I_{it}^{-}$
- Allow I_{it} to become positive or negative
- Penalize I_{it}^{+} , I_{it}^{-} differently in objective if desired

$$max \ \sum_{t=1}^{\bar{t}} \left\{ \sum_{i=1}^{m} r_i S_{it} - h_i I_{it}^{+} - \pi_i I_{it}^{-} \right\}$$

Overtime:

- Define O_{jt} as hours of OT used on resource j in period t and β_j the cost of one overtime hour in workstation j
- Add O_{jt} to c_{jt} in capacity constraint.

$$\sum_{i=1}^{m} a_{ij} X_{it} \leq c_{jt} + O_{jt} \quad for\ all\ j, t$$

- Penalize O_{jt} in objective if desired

$$max \ \sum_{t=1}^{\bar{t}} \left\{ \sum_{i=1}^{m} r_i S_{it} - h_i I_{it}^{+} - \pi_i I_{it}^{-} - \sum_{j=1}^{n} \beta_j O_{jt} \right\}$$

Workforce Planning

- **Problem:** determine most profitable production and hiring/firing policy over planning horizon.
- **Motivation for Study:**
  - hiring/firing vs. overtime vs. Inventory Build tradeoff
  - iterative nature of optimization modeling.
- **Inputs:**
  - demand forecast (assume single product for simplicity)
  - unit hour data
  - labor content data
  - capacity constraints
  - hiring/ firing costs
  - overtime costs
  - holding costs
  - unit profit

A Workforce Planning Model (I)

Notation

j = an index of workstation, $j = 1, \ldots, n$

t = an index of period, $t = 1, \ldots, \bar{t}$

$\bar{d}_t$ = maximum demand in period t.

$\underline{d}_t$ = minimum sales allowed in period t

a_j = unit hours on workstation j

b = number of man hours required to produce one unit.

c_{jt} = capacity of work center j in period t.

r = net profit from one unit.

h = cost to hold one unit for one period t.

l = cost of regular time in dollars / man - hour

l' = cost of overtime in dollars / man - hour

e = cost to increase workforce by one man - hour

e' = cost to decrease workforce by one man - hour

A Workforce Planning Model (II)

Notation (cont.)

X_t = amount produced in period t

S_t = amount sold in period t

I_t = inventory at end of t

W_t = workforce period t in man-hours of regular time

H_t = increase (hires) in workforce from period $t-1$ to t in man-hours.

F_t = decrease (fires) in workforce from period $t-1$ to t in man-hours.

O_t = overtime in period t in hours

Note, this model only considers a single product. Generalizations to m products are straightforward!

A Workforce Planning Model (III)

Formulation

$$\max \sum_{t=1}^{\bar{t}} \left\{ r S_t - h I_t - l W_t - l' O_t - e H_t - e' F_t \right\}$$

subject to

$$\underline{d}_t \le S_t \le d_t \qquad \text{for all } t$$

$$a_j X_t \le c_{jt} \qquad \text{for all } t$$

$$I_t = I_{t-1} + X_t - S_t, \quad \text{for all } t$$

$$W_t = W_{t-1} + H_t - F_t \quad \text{for all } t$$

$$b X_t \le W_t + O_t \qquad \text{for all } t$$

$$X_t,\ S_t,\ I_t,\ O_t,\ W_t,\ H_t, F_t \ge 0 \quad \text{for all } t$$

A Workforce Planning Example (I)

Problem Description

- 12 month planning horizon
- 168 hours per month
- 15 workers currently in system
- regular time labor at $35 per hour
- overtime labor at $52.50 per hour
- $2,500 to hire and train new worker

  $2,500/168=$14.88 ≈ $15/hour
- $1,500 to lay off worker

  $1,500/168=$8.93 ≈ $9/hour
- 12 hours labor per unit
- demand assumed met ($S_t = d_t$, *so* S_t variables are unnecessary)

A Workforce Planning Example (II)

- **Solution:**
  - LP optimal Solution: layoff 9.5 workers
  - Add constraint: $F_t=0$
    - results in 48 hours/worker/week of overtime
  - Add constraint: $O_t \leq 0.2W_t$
    - Reasonable solution?

Aggregate Planning Conclusions

- No single AP model is right for every situation
- Simplicity promotes understanding
- Linear programming is a useful AP tool
- Robustness matters more than precision
- Formulation and Solution are *not* separate activities.